AP Calculus BC Unit 5: Free Response Questions

1) a) $f''(x) = x^2e^{2-x} - 1 = 0$ @ $x = 0.463$ & $x = 5.356$

f(x) is concave down on $(0, 0.463)$ & $(5.356, 6)$

b) $f'(2) = 0$

$f''(2) = 3 > 0$

f(x) has a relative min @ $x = 2$

c) $f''(c) = \frac{f'(4) - f'(2)}{4-2} = \frac{f'(4) - 0}{2}$

$f''(c) = \frac{8.5}{2} = 4.25$

$f''(c) \neq 4.25$ on $(2, 4)$ so $f'(4) \neq 8.5$

d) $f''(5) = 25e^{-3} - 1 \neq 0$

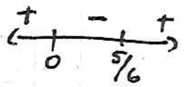
f(x) does not have a point of inflection at $x = 5$.

2) a) $f'(x) = (6x^2 - 5x)e^x$

$6x^2 - 5x = 0$

$x(6x - 5) = 0$

$x = 0$ $x = 5/6$



f has a max @ $x = 0$ b/c f' Δs signs from + to −

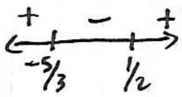
f has a min @ $x = 5/6$ b/c f' Δs signs from − to +

b) $f''(x) = (6x^2 - 5x)e^x + (12x - 5)e^x$

$= e^x(6x^2 + 7x - 5) = 0$

$(3x + 5)(2x - 1)$

$x = -\frac{5}{3}$ $x = 1/2$



f has P.o.I @ $x = -5/3$ & $x = 1/2$ b/c f'' Δs signs.

c) g' Neg; Neg ← g is dec

g'' Neg ← g is concave down

Pos ← g is concave up

d) $h(x) = f(x)g(x)$

$h'(x) = f(x)g'(x) + g(x)f'(x)$

ON $(1, 5)$:

$h'(x) = (+)(-) + (-)(+) < 0$

h(x) is decreasing

3) $F(t) = 10(t^3-12t^2+45t+100)$

a) $F'(4) = -30$

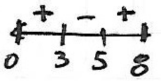
At $t=4$, the number of fish in the bay is decreasing at a rate of 30 fish/day.

b) $F'(t) = 10(3t^2-24t+45) = 0$

$(t^2-8t+15) = 0$

$(t-5)(t-3) = 0$

$t=5 \quad t=3$



$F(0) = 1000$

$F(3) > 1000$

$F(5) = 10(125-300+225+100) = 1500$

$F(8) > F(5)$

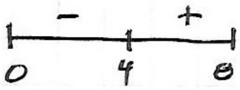
The abs. min # of fish in the bay is 1000 @ t=0.

c) rate of change is decreasing → $F'(t)$ is decreasing

$F'' < 0$

$F''(t) = 10(6t-24) = 0$

$t = 4$



on $[0,4]$, $F''(t) < 0$ therefore the rate of change of the number of fish in the bay is decreasing

d) $P(F(t))$ ← # of pelicans

$P'(F(t)) \cdot F'(t)$ ← rate of change of the # of pelicans

$P'(F(c)) \cdot F'(c) = P'(10(c^3-12c^2+45c+100)) \cdot (10(3c^2-24c+45))$